Internet Robustness Metrics
Complex Network Metrics Analysis

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Abstract
Many robustness and resilience metrics for graphs have been proposed. But the notion of robustness/resilience is still unclear. Currently, there are two notions of robustness: *Classical*, which relies on basic graph theory concepts; *Contemporary* where services are supported by the networks. This research aims to gather these metrics and find applications to the Internet computer network. We find many robustness/resilience metrics in different areas, such as Communication, Service-Oriented, Social and Biological(Cancer) networks, where its applicability on The Internet are still not studied. Furthermore, we intent to apply these results in the Chilean Internet network.

Introduction

To achieve an unbiased research about the broad topic which is robustness in complex networks, we perform a *Systematic Mapping Study* [5].

The main question is **What metrics to study graph robustness do exist?**, its purpose is to get full knowledge about network resilience metrics and measures.

The next table extracted from [4], describe some of these metrics classified by its approach. The classical approach are based on graph theory concepts such as connectivity, clique, degree and shortest path. The contemporary approach, as stated in [1] is the ability of a network to maintain its total throughput (or degrade gracefully) under node and link removal, taking into account services provided by the network.

Approach	Characteristic
Classical	Average Nodal degree ($\langle k \rangle$)
	Node connectivity (κ)
	Link connectivity (ρ)
	Heterogeneity ($\sqrt{\sigma_k^2}/\langle k \rangle$)
	Symmetry ratio ($\epsilon/(D+1)$)
	Diameter (D)
	Average shortest-path length ($\langle l \rangle$)
	Assortativity coefficient (r)
	Average neighbor connectivity ($\frac{k_{nn}}{\lvert v \rvert - 1}$)
	Clustering coefficient ($\langle C \rangle$)
	Betweenness centrality ($\langle b \rangle$)
	Algebraic connectivity ($\mu_{\lvert v \rvert - 1}$)
	Average two-Terminal Reliability (A2TR)
Contemporary	Elasticity (E)
	Quantitative Robustness Metric (QNRM)
	Qualitative Robustness Metric (QLRM)
	R-value (R)
	Viral Conductance (VC)

Table 1: Classical and Contemporary Robustness Metrics

Research

Specifying the research questions is the most important part of any systematic review. The review questions drive the entire systematic review methodology. The next diagram describes the research protocol.

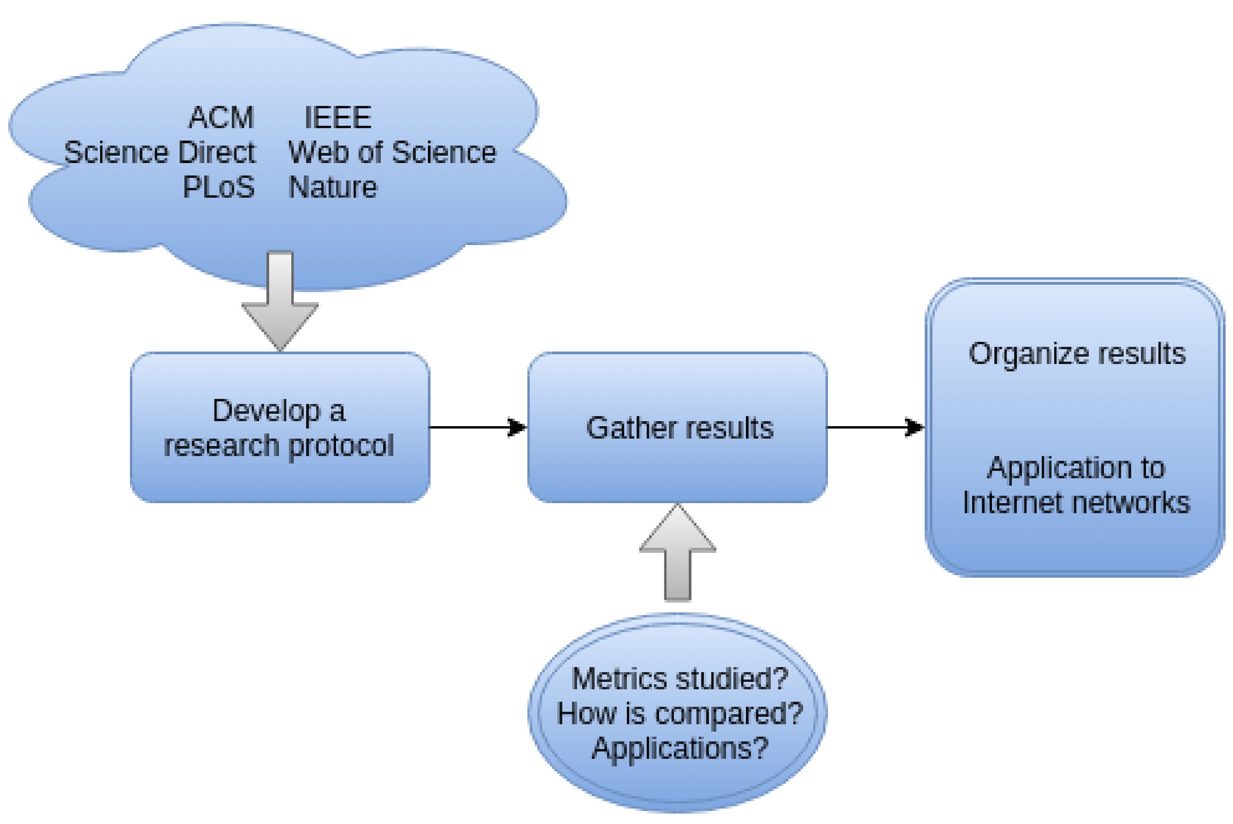

Figure 1: Protocol Diagram

Strategy

The search keywords and operators are shown in the next diagram. The operation $X\ PRE/n\ Y$ means that X word precedes the Y word distanced by n words. Stemming processes of the search engines are considered. We search these terms in the title and abstract section (not necessarily both).

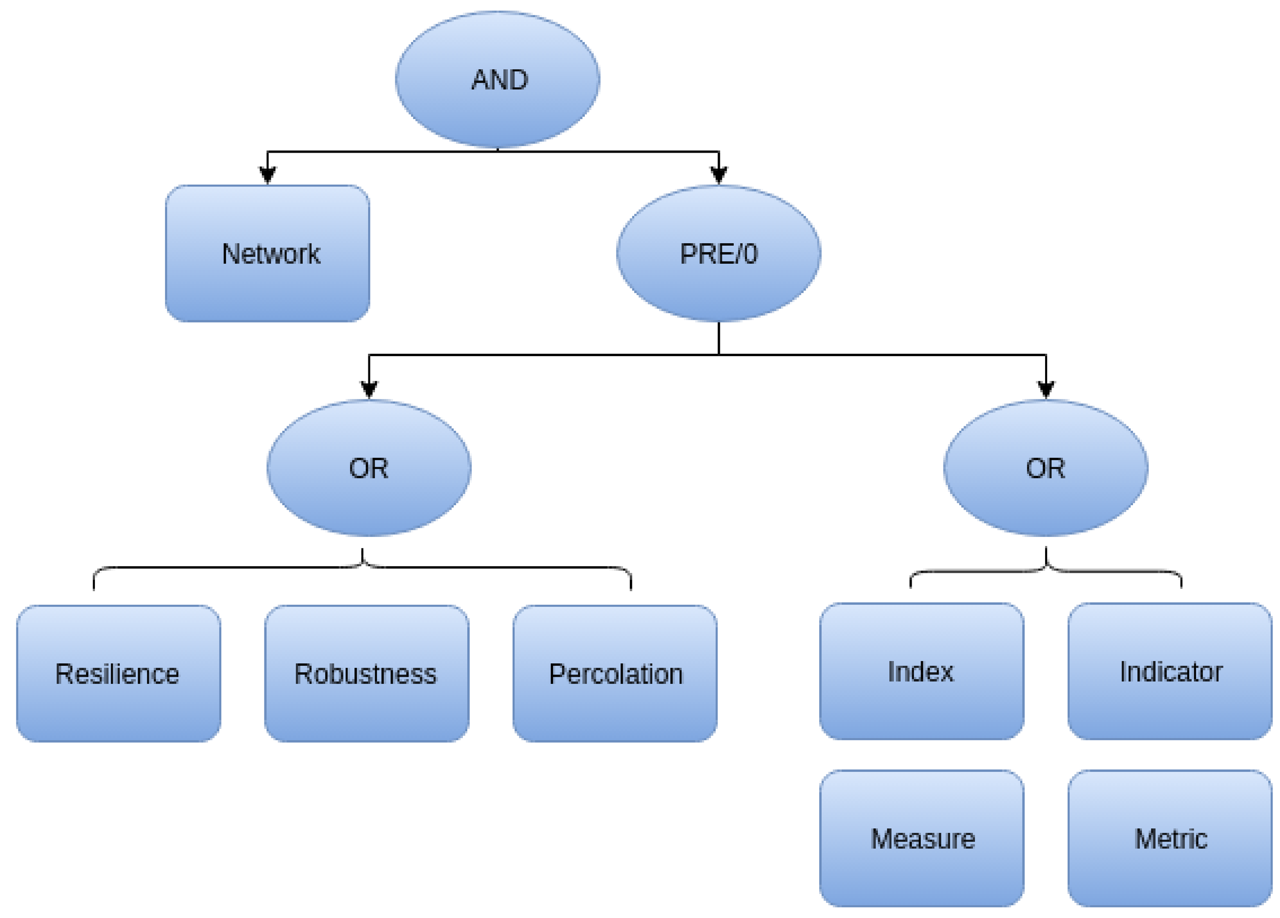

Figure 2: Search Diagram

Results

Around 300 articles were pre-selected where 200 passed the first filter (is primary study, is in English, is about complex networks and others). Several new metrics have been found, which many have the potential to be applied on The Internet. Some of them were proposed in different topics and then enhanced to be applied to a more general topic. An example of this is the *Estrada Index* (S), proposed by Estrada [2], a topological index of protein folding. Then Jun Wu *et al.* [3] proposed a variation called *Natural Connectivity* ($\bar{\lambda}$), which has much broader applications.

$$S = \sum_{j=1}^{N} e^{\lambda_j}$$

$$\bar{\lambda} = ln(S/N) = ln\left[\frac{1}{N}\sum_{j=1}^{N} e^{\lambda_j}\right]$$

References

[1] P. Schumm R. E. Kooij A. Sydney, C. Scoglio. *ELASTICITY: Topological Characterization of Robustness in Complex Networks*. 2008.

[2] E. Estrada. *Characterization of 3D molecular structure*. Mar. 2000.

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[4] Samee U. Khan Eusebi Calle Keqin Li Kashif Bilal, Marc Manzano and Albert Y. Zomaya. *On the Characterization of the Structural Robustness of Data Center Networks*. Jan-Jun. 2013.

[5] Mark Petticrew and Helen Roberts. *Systematic Reviews in the Social Sciences: A Practical Guide*. 2005.